

1. ORIGINATING ACTIVITY (Corporate author)	2. REPORT SECURITY CLASSIFICATION
Aerospace Medical Research Laboratory	Unclassified
Aerospace Medical Div, Air Force Systems Command	3. GROUP
Wright-Patterson Air Force Base, Ohio 45433	N/A

3. REPORT TITLE
USE OF A MATHEMATICAL MODEL FOR THE EVALUATION OF HEAD INJURY CRITERIA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (First name, middle initial, last name)
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6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1971	16	9

8a. CONTRACT OR GRANT NO.	8b. ORIGINATOR'S REPORT NUMBER(S)
a. PROJECT NO. 7231	AMRL-TR-71-29
c.	Paper No 4
d.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT
Approved for public release; distribution unlimited

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Aerospace Medical Research Laboratory Aerospace Medical Div, Air Force Systems Command, Wright-Patterson AFB, OH 45433

13. ABSTRACT
The Symposium on Biodynamics Models and Their Applications took place in Dayton, Ohio, on 26-28 October 1970 under the sponsorship of the National Academy of Sciences - National Research Council, Committee on Hearing, Bioacoustics, and Biomechanics; the National Aeronautics and Space Administration; and the Aerospace Medical Research Laboratory, Aerospace Medical Division, United States Air Force. Most technical areas discussed included application of biodynamic models for the establishment of environmental exposure limits, models for interpretation of animal, dummy, and operational experiments, mechanical characterization of living tissue and isolated organs, models to describe man's response to impact, blast, and acoustic energy, and performance in biodynamic environments.

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PAPER NO. 4

USE OF A MATHEMATICAL MODEL FOR
THE EVALUATION OF HEAD INJURY CRITERIA

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ABSTRACT

The present study concerns the use of an analytic model for closed brain injury to evaluate General Motors' Severity Index for head injury. This index, which considers the relative importance of the duration and intensity of the pulse by means of an impulse-integration procedure, has been demonstrated empirically to be an excellent means of comparing the severity of pulses of varying shapes. Four pulses (square, half-sine, skewed and triangular), each having the same Severity Index but different magnitudes, were employed as inputs to the model.

The model response to each of the four impacts was determined. Graphs of the time variation of stresses in the shell and pressures in the fluid at both the impact pole and at the counter pole demonstrate excellent correlation between model response and Severity Index, independent of the shape and magnitude of the impact force.

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Criteria to assess the severity of impacts classically have been based upon specific parameters of the pulse wave-form. Maximum force or acceleration sustained, rate of onset, pulse shape, duration, and the rate of change of acceleration are representative of characteristics of the pulse which have been selected as injury criteria. However, as von Gierke¹ has noted, the situations in which any single aspect of the impact wave-form characterizes the system response are limited. This observation is substantiated by the work of Kornhauser and Gold² who have found that the incidence of injury depends on both the magnitude and the duration of the impact. Their tolerance curves of velocity change (Δv) versus average acceleration (\bar{a}) indicate that threshold values for both Δv and \bar{a} must be exceeded before injury will occur. Thus, a relation between both magnitude and duration must be considered in assessing impact severity.

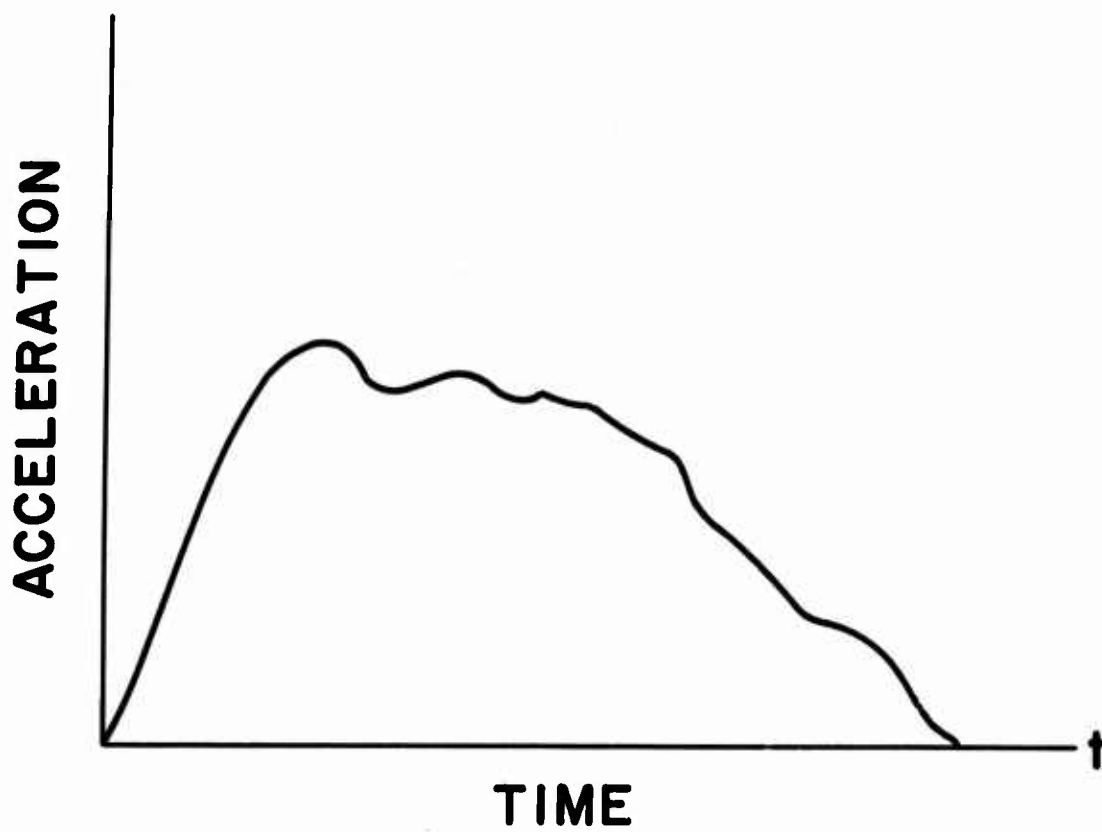
Based on the assumption that the severity of an impulse wave-form is a function of both duration and intensity, Gadd³ developed an exponentially weighted-impulse criterion for estimating head injury resulting from frontal impacts. From the work of Lissner^{4,5} in which tolerance curves of intracranial pressure versus time indicated that the time required to produce severe concussion in dogs decreased as the pressure increased, and from that of Eiband⁶ which demonstrated a similar trend for sled-test tolerance curves of acceleration versus time, Gadd noted that both results demonstrated a downward sloping tolerance curve in the range of vehicle occupant cranial impacts. He further noted that when the data was represented on a log-log plot the curve could be approximated by a straight line at an angle of less than 45° with the horizontal. He concluded that an impulse-area criterion would not satisfactorily predict injury. The resulting severity index which Gadd devised is depicted in Figure 1.

In general, the severity index (S. I.) may be expressed mathematically as:

$$S. I. = \int (\bar{a})^n dt$$

where S. I. is the severity index (a number); \bar{a} is the acceleration, pressure, or force producing an injury threshold of a specified degree; n is the exponential weighting factor; and t is time in seconds.

For head injury resulting from frontal impacts, \bar{a} is the acceleration of the head expressed in "g" units, and n, based upon empirical results, is 2.5. The exponential weighting factor signifies the importance of the high intensity segments and the insignificance of the low intensity portions of the impacts, while the integral accounts for the



$$S.I. = \int_0^t a^{2.5} dt$$

Figure 1. General Motors Severity Index

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effect of the duration of both the high intensity and the low intensity portions of the curve.

The purpose of this study is to investigate the usefulness of the severity index in comparing the severity of pulses of different peak magnitude and varying wave-form. The severity index has been used primarily to investigate the injury potential of various pulses of one to fifty milliseconds duration applied to the frontal bone of the head. A previously developed mathematical model for head injury⁷ has been used therefore to study various pulses having the same severity index but differing in wave-form and peak magnitude.

The head injury model assumes that the skull is a linear, homogeneous, isotropic, thin spherical shell and that the brain is an ideal (acoustic) fluid. Using extensional shell theory and assuming axisymmetry of the load, the following set of three coupled, second order, linear partial differential equations with variable coefficients obtain in spherical polar coordinates:

$$\begin{aligned} \frac{\partial^2 u}{\partial \phi^2} + \cotan \phi \frac{\partial u}{\partial \phi} - (\mu + \cotan^2 \phi) u + (1 + \mu) \frac{\partial w}{\partial \phi} \\ - \lambda^2 \frac{\partial^2 u}{\partial t^2} = 0, \end{aligned} \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial \phi} + \cotan \phi u + 2w + \frac{\lambda^2}{(1 + \mu)} \frac{\partial^2 w}{\partial t^2} + \frac{\eta \lambda^2}{(1 + \mu)} \left(\frac{\partial \Phi}{\partial t} \right)_R \\ - \frac{\epsilon (1 - \mu)}{E} F_{\text{ext}} = 0, \end{aligned} \quad (2)$$

and

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \phi^2} + \frac{\cotan \phi}{r^2} \frac{\partial \Phi}{\partial \phi} - \frac{\partial^2 \Phi}{\partial t^2} = 0 \quad (3)$$

Figure 2 depicts the relationship between the displacements u , v , and w and the coordinate axes.

Equations (1) - (3) are solved by finite difference methods using backwards difference analogs and employing techniques described in references 7 and 8. Figure 3 shows the segmentation of the shell for

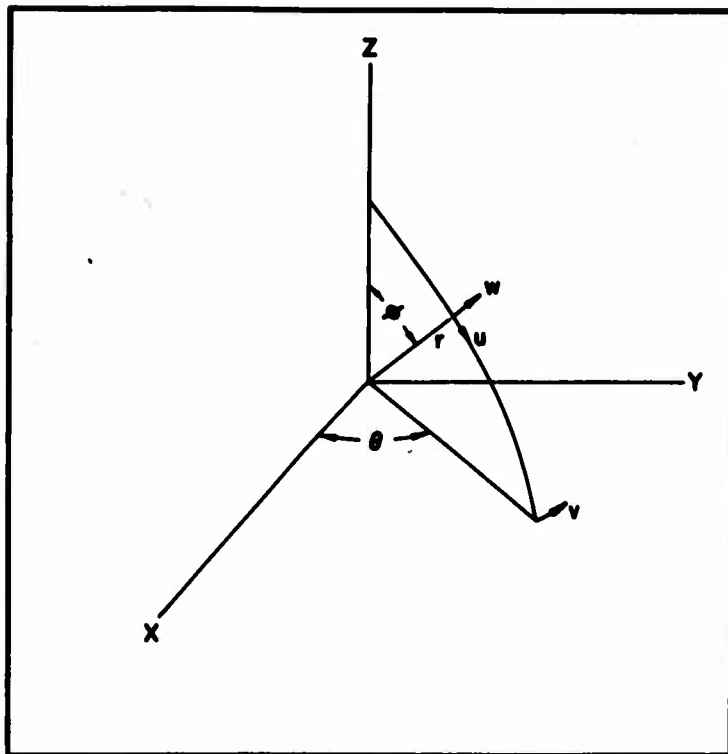


Figure 2. Coordinate System used to Describe the Model.

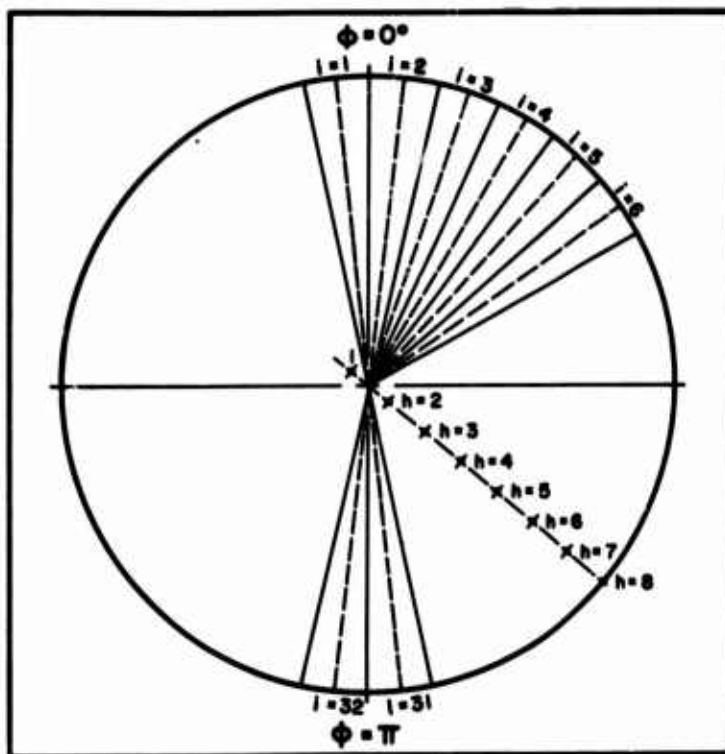


Figure 3. Shell segmentation for Finite Difference Analogs.

application of the finite difference analogs.

The model may be applied to an input with arbitrary temporal and spatial distributions, provided axisymmetry is maintained. Four different inputs were selected to evaluate the severity index, and the model responses for each have been compared. The four acceleration pulse shapes chosen to represent those which might be encountered in actual vehicle-interior impact situations are square pulse, half-sine pulse, triangular pulse and an arbitrary impact pulse as suggested by Goldsmith⁹. These four acceleration pulses are depicted schematically in Figure 4. The mathematical representations of the four pulses are shown in Figure 5, in which t is the independent variable, time, and T is the duration of the impact.

A pulse duration of 0.0001 sec was selected for each of the four input functions. The intensity of the pulses was selected by requiring that each of the four pulses yields the same severity index. That is,

$$(S. I.)_i = \int (\bar{a})_i^{2.5} dt' = 500.$$

The input to the model, represented by the term F_{ext} in equation (2), has units of lbf/in^2 . Each of the four input pulses was expressed in these units by assuming a head weight of 10 pounds and an even distribution of the load over a polar cap of 15° polar angle. Figure 6 depicts the four inputs as functions of both time and polar angle.

Numerical Results

The following properties of the shell and the encased fluid are used in the calculations:

Shell:	$E = 2 \times 10^6 \text{ lbf/in}^2$
	$\mu = 0.25$
	$a = 3 \text{ in.}$
	$h = 0.15 \text{ in.}$
	$\rho_s = 2.00 \times 10^{-4} \text{ lb sec}^2/\text{in}^4$

ACCELERATION PROFILES

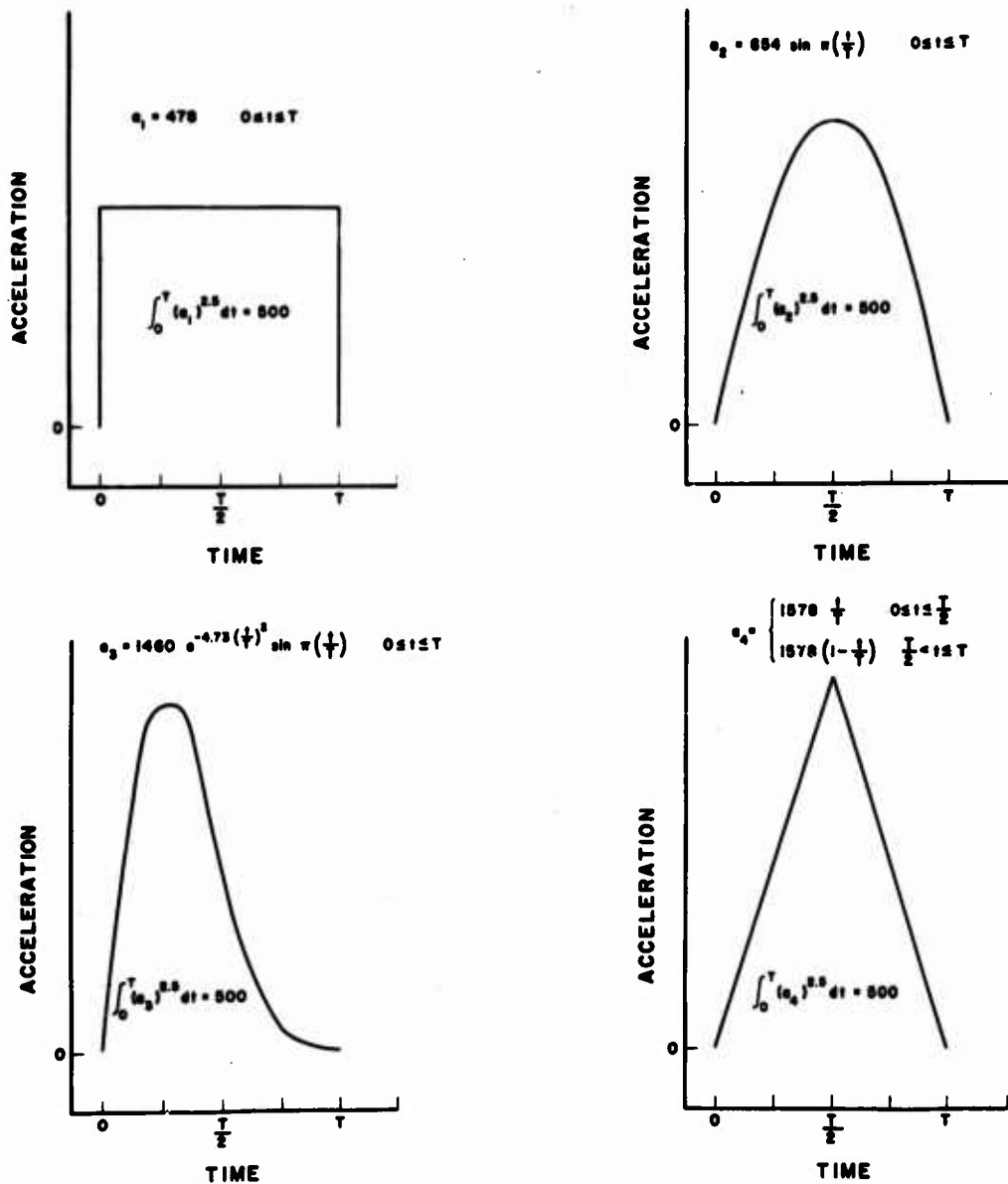


Figure 4. Schematic Representation of the Four Input Pulses.

TYPE OF PULSE	MATHEMATICAL DESCRIPTION	TIME DOMAIN
SQUARE	$a_1 = A_1$	$0 \leq t \leq T$
HALF-SINE	$a_2 = A_2 \sin \frac{\pi t}{T}$	$0 \leq t \leq T$
ARBITRARY	$a_3 = A_3 e^{-b\left(\frac{t}{T}\right)^2} \sin \frac{\pi t}{T}$	$0 \leq t \leq T$
TRIANGULAR	$a_4 = A_4 \frac{t}{T}$ $a_4 = A_4 \left(1 - \frac{t}{T}\right)$	$0 \leq t \leq \frac{T}{2}$ $\frac{T}{2} < t \leq T$

Figure 5. Mathematical Description of the Four Input Pulses.

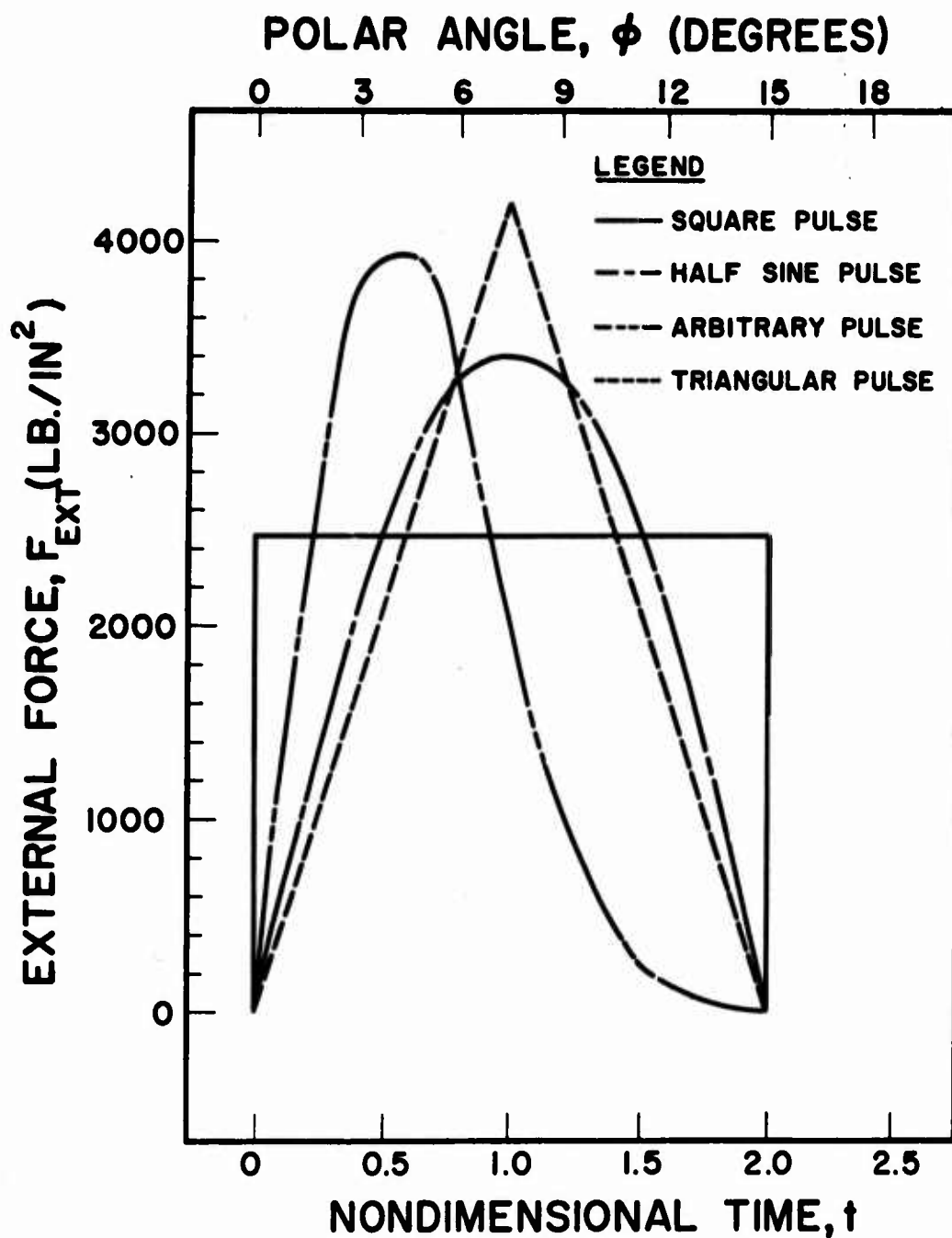


Figure 6. Forcing function versus time, t , and polar angle, ϕ .

Fluid:	$K = 300,000 \text{ lbf/in}^2$
	$\rho_f = 0.938 \times 10^{-4} \text{ lb sec}^2/\text{in}^4$
	$C_o = 56,553 \text{ in/sec.}$

Using this data and determining the model response to each of the four input pulses yields the results shown in Figures 7 - 11. The resting position is considered to be a zero datum, and all pressures are gage rather than absolute pressures.

Pressure variation with time at polar angles of 3° and 177° , the positions approximating the impact pole and the counter pole, respectively, are shown in Figures 7 and 8. Fluid pressures are studied at $r = 1.0$ (immediately adjacent to the shell) since this location is where the greatest pressure fluctuations occur. The fluid at the impact pole is compressed initially and as the impact load decreases in intensity the compression diminishes and the fluid is subjected to a tensile stress. The duration of this tensile stress is approximately $125 \mu\text{sec}$. The pressure at the counterpole does not reflect a change from the zero datum until the stress wave in the shell, which generates instantaneous centers for wave propagation as it traverses the shell, approaches the counterpole. As the shell attempts to separate from the fluid, tensile stresses occur in the fluid. These stresses are more diffuse, less severe and longer lasting than the tensile stresses at the impact pole. Although the solution has not been obtained for a period that is sufficiently long to determine the duration accurately, it is estimated that the tensile stress at the counterpole is maintained for approximately $300 \mu\text{sec}$.

The variation of the shell radial displacement as a function of time for both the impact pole and the counterpole are shown in Figures 9 and 10. The load produces an initial indentation at the impact pole followed by an elastic recovery as the load is removed. There is a continual outward displacement at the counterpole which assists in producing the tensile stresses in the fluid at that point. Membrane stresses in the shell are shown in Figure 11 for the impact pole. Initially high compressive stresses are generated during the impact period, followed by elastic recovery when the load is relieved.

Each of Figures 7 - 11 shows the model response for the four impact pulses selected. Inspection of these five graphs reveals a similar model response for the four pulses although the pulse shapes, rates of onset, and peak intensities differ.

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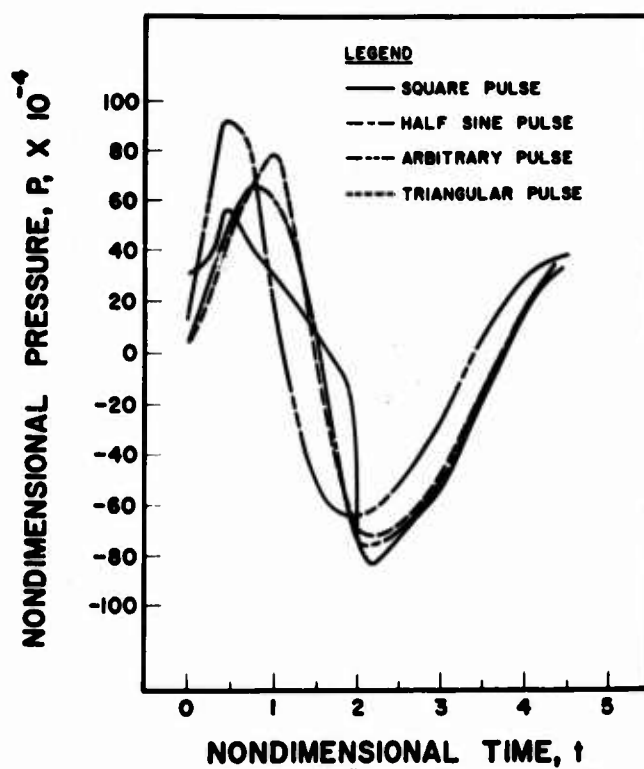


Figure 7. Pressure versus time, $\phi = 3^\circ$, $r = 1.0$

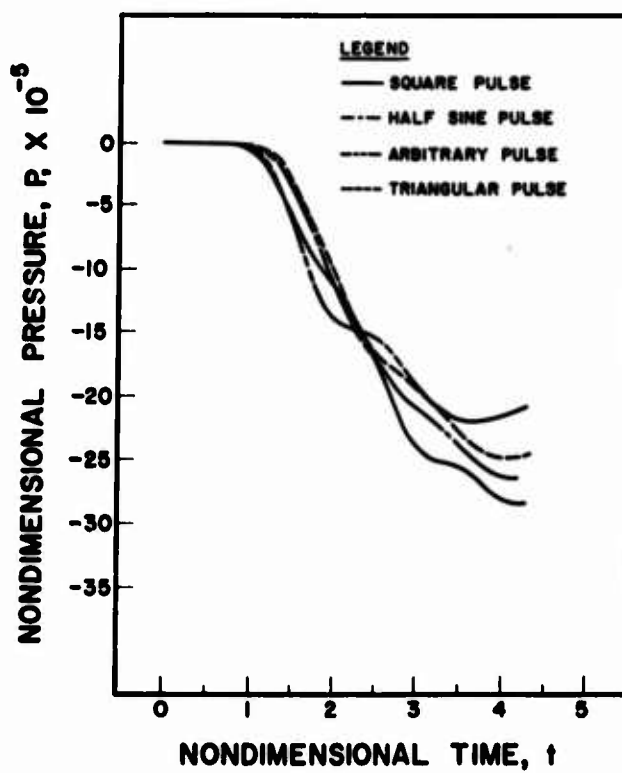


Figure 8. Pressure versus time, $\phi = 177^\circ$, $r = 1.0$

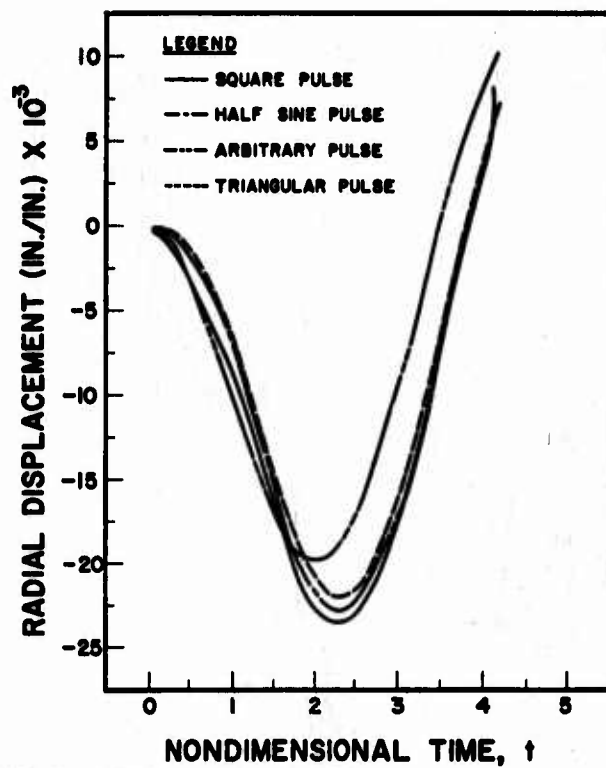


Figure 9. Radial displacement versus time, $\phi = 3^\circ$

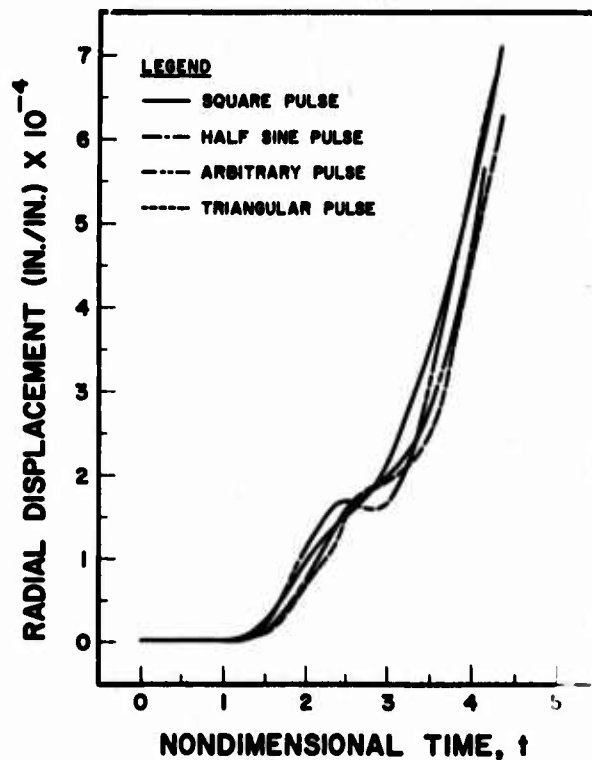


Figure 10. Radial displacement versus time, $\phi = 177^\circ$

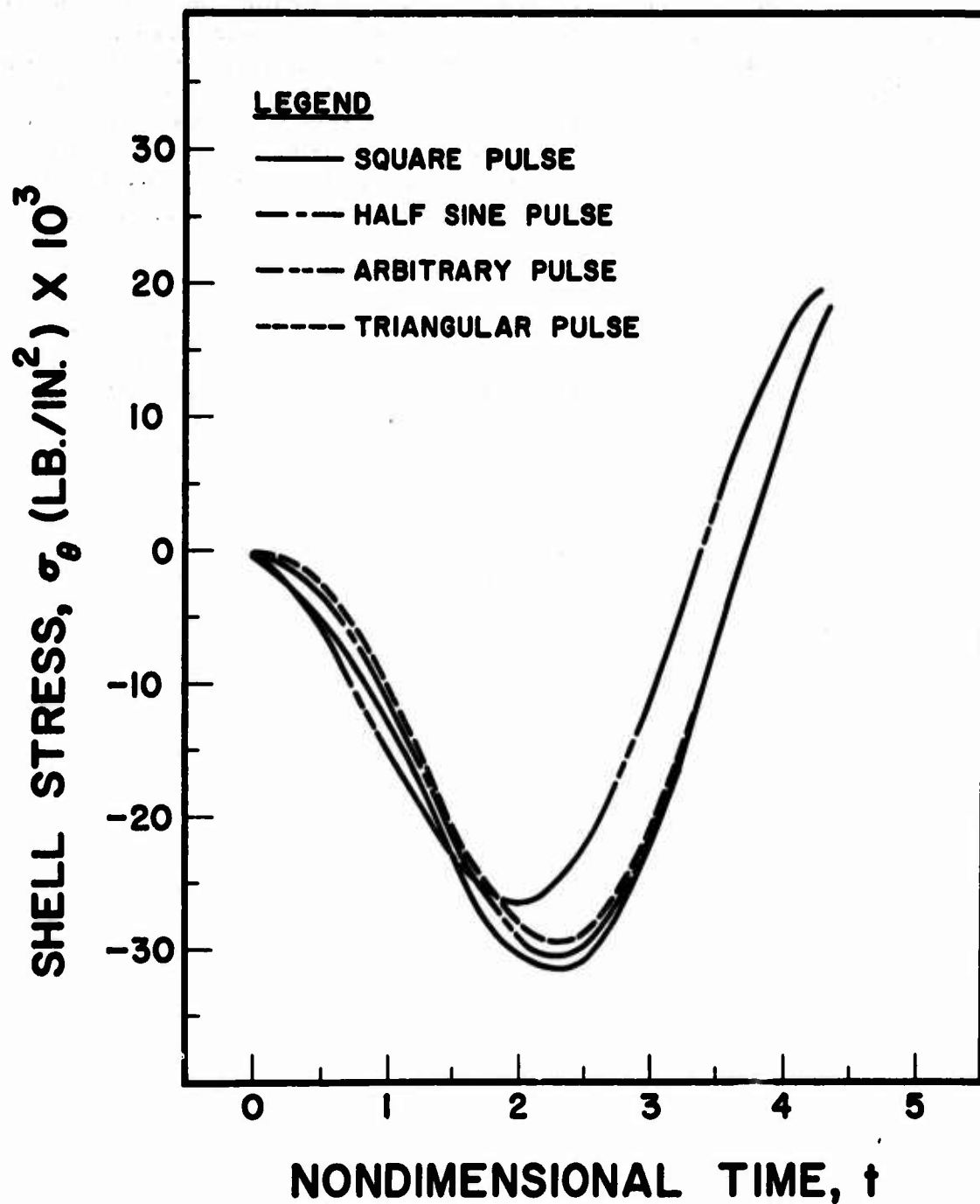


Figure 11. Normal stress in the θ -direction versus time, $\phi = 3^\circ$

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Conclusions

The intent of this study has not been to determine the effectiveness of the General Motors' Severity Index in predicting injury thresholds, since this result could be accomplished only in a program which combined analytic and experimental or clinical investigation. The objective of this study has been to assess the value of the Severity Index in predicting a single parameter equivalent for pulse wave-forms of varying shapes and magnitudes. As evidenced by the results shown in Figures 7 - 11, the Severity Index does appear to be an excellent means of comparing impact functions.

The current work may be extended to an investigation of model response to pulses of durations within the range of known vehicle interior impacts, or to a study of pulses of identical peak intensities but different durations. The predictive capability of the severity index under these conditions could therefore be determined.

Acknowledgments

The author expresses his appreciation to the Life Sciences Division of Technology Incorporated for support of this study.

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NOMENCLATURE

a	Radius of the shell middle surface (L)
\bar{a}	Average acceleration
A_1, A_2, A_3, A_4	Constants, determine magnitude of the acceleration functions
b	Constant, determines decrement rate of the loading function
C_o	Compressional wave speed in the encased fluid, $[K/\rho_o]^{1/2}$ (L/T)
C_s	Wave speed in the shell, $[E/\rho_s(1 - \mu^2)]^{1/2}$ (L/T)
E	Young's modulus for the shell material (F/L ²)
F_{ext}	External loading function (F/L ²)
h	Shell thickness (L)
K	Bulk modulus of the fluid (F/L ²)
n	Exponential weighting factor
$P(\phi, t)$	General loading function (F/L ²)
P'	Pressure in the fluid measured with the undisturbed state as zero datum (F/L ²)
P	Nondimensional pressure, P'/K
R	Value of the discrete radial variable at the shell boundary
SI	Severity index
r'	Dimensional radial coordinate (L)
r	Nondimensional radial coordinate, r'/a
T	Nondimensional duration of loading
t'	Dimensional time (T)
t	Nondimensional time, $C_o t'/a$

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u'	Dimensional meridional displacement (L)
u	Nondimensional meridional displacement, u'/a
v'	Dimensional parallel displacement (L)
v	Nondimensional parallel displacement, v'/a
w'	Dimensional radial displacement (L)
w	Nondimensional radial displacement, w'/a
x, y, z	Cartesian coordinates
Δv	Incremental change in velocity
ϵ	Dimensionless parameter, a/h
η	Dimensionless parameter, $\rho_o/h\rho_s$
ϕ, θ	Spherical coordinates
λ	Dimensionless parameter, C_o/C_s , the speed ratio
μ	Poisson's ratio
ρ_f	Density of the fluid (FT^2/L^4)
ρ_s	Density of the shell (FT^2/L^4)
ψ	Dimensional velocity potential (L^2/T)
Φ	Nondimensional velocity potential, ψ/aC_o
Subscripts	
h	Refers to discrete variables in the radial direction
i	Refers to discrete variables in the ϕ direction

REFERENCES

1. von Gierke, H. E., "Biomechanics of Impact Injury." Impact Acceleration Stress - Proceedings of a Symposium with a Chronological Bibliography. National Academy of Science, National Research Council Publication 977, 1962, pp. 121-122.
2. Kornhauser, M. and A. Gold, "Application of the Impact Sensitivity Method to Animate Structures." Impact Acceleration Stress - Proceedings of a Symposium with a Comprehensive Chronological Bibliography. National Academy of Sciences, National Research Council Publication 977, 1962, pp. 333-344.
3. Gadd, C. W., "Use of a Weighted Impulse Criterion for Estimating Injury Hazard." Proceedings of Tenth Stapp Car Crash Conference, 1966. Published by SAE, New York. pp. 95-100.
4. Lissner, H. R., et. al., "Experimental Studies on the Relation between Acceleration and Intracranial Pressure Changes in Man." Surgery, Gynecology and Obstetrics, 1960, pp. 329-338.
5. Lissner, H. R. and E. S. Gurdjian, "Experimental Cerebral Concussion." ASME Paper No. 60-WA-273, 1960.
6. Eiband, A., "Human Tolerance to Rapidly Applied Accelerations." NASA Memorandum 5-19-59E, June 1959.
7. Benedict, J. V., E. H. Harris and D. U. von Rosenberg, "An Analytical Investigation of the Cavitation Hypothesis of Brain Damage." J. Basic Engineering - Transactions of the ASME, Vol. 92, Series D, No. 3, September 1970, pp. 597-603.
8. von Rosenberg, D. U., Methods for the Numerical Solution of Partial Differential Equations, New York. American Elsevier, 1969, p. 128.
9. Goldsmith, W., Private Communication, 1968.